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Universality classes for the ricepile model with absorbing properties

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The absorbing ''ricepile'' model with stochastic toppling rules has been numerically studied. Local limited, local unlimited, nonlocal limited, and nonlocal unlimited versions of the absorbing model have been investigated. Transport properties and different dynamical regimes of all of the models have been analyzed, from the point of view of self-organized criticality. Phase transitions between different dynamical regimes were studied in detail. It was shown that the absorbing models belong to two different universality classes.

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I. INTRODUCTION

Self-organized criticality (SOC) has been a widely studied phenomenon in the past ten years. The theory of SOC, proposed by Bak, Tang, and Wiesenfeld [1], describes the dynamical behavior of many-particle systems with local interactions. Paradigmatically, the description is based on the dynamics of a pile of sand. If the sandpile is randomly driven by the slow addition of sandgrains, the slope of the pile increases and after some time local stability conditions are violated somewhere on the pile surface. The avalanche starts to slide down the slope. This type of dynamics is easily modeled by a cellular automaton. Such a model ''sandpile'' is defined on a large n-dimensional lattice. The avalanche dynamics is, under the action of slow drive, governed by the local critical conditions (such as the local critical slope, for example) and local toppling rules. This dynamics leads to the steady state called the self-organized critical state, characterized by the critical scaling of the avalanche size distribution:

$$p(s,L)=s^{-\tau}f\left(\frac{s}{L^D}\right). \tag{1}$$

In Eq. (1), L is the system size and s exhibits the size of the avalanche. The critical exponents τ and D depend on the particular model. Different models can be divided into the various universality classes, defined by a specific set of critical exponents [8,10].

A natural step from the sandpile model systems leads to the investigation of real piles of granular material from the point of view of SOC dynamics. Several efforts have been made in this direction [2–4], but no clear evidence of self-organized critical dynamics (1) has been found. Finally, in 1996 an experiment was done by a group of experimentalists and theoretists in Oslo [5]. In the Oslo experiment, dynamical behavior of the driven quasi-one-dimensional pile of rice was investigated. The avalanche sizes in the steady state were measured in terms of dissipated potential energy. Two types of grains were used (elongated and round ones), showing completely different dynamics. In the case of the ricepile, consisting of elongated grains, the SOC state has been established, in which the avalanche size distribution has a power-law character with critical exponent $\tau\approx 2.02$ [5].

Soon after the experimental results were published, the model ''ricepile'' cellular automata were suggested and numerically studied [6,7–9]. The model ricepile is, in principle, a cellular automaton defined on a one-dimensional lattice, with randomness incorporated into the toppling rules and with deterministic drive. Changes in toppling rules are often manifested by different dynamical behavior of the model and a different universality class in which the model belongs [9,10].

In this paper, we study a model that exhibits a modification of the two-threshold ricepile model [6,7]. In [6,7], the gravity effects, grain friction, and the local conditions on the pile are described by the parameter p. Two thresholds, namely the critical threshold and the gravity threshold, are defined. Thresholds governs the movement of the grain on the pile surface.

We removed the second, gravity threshold of the ricepile model. Due to this, depending on the parameter p value, the mode has absorbing properties and interesting dynamical regimes [11]. The one-threshold model in two dimensions has been studied by Tadić and Dhar [12].

Here, different versions of the absorbing model are defined using different toppling rules. The dynamical behavior of the local limited (LLIM), local unlimited (LUNLIM), nonlocal limited (NLIM), and nonlocal unlimited (NUNLIM) absorbing model is studied and two distinct universality classes are recognized.

II. RICEPILE MODELS

The experimental results of the Oslo group motivated further theoretical studies of the avalanches and dynamics of granular material [13,14]. The main question is, which physical properties of the pile granular material are important for the SOC state to be established? What is, for example, the role of friction, what is the role of the grain shape, and what is the role of the gravity and inertia of rolling grains? The ricepile experiment reveals that no SOC is possible if the ricepile consists of round ricegrains. On the contrary, the dynamics of pile consisting of elongated grains is self-organized critical [5]. Certainly, the shape of the grains and the additional effects related to the shape, such as better packing of grains due to the elongated shape, suppressed rolling of grains, and thus suppressed inertia effects, are of great importance [5,14].

Ricepile models are cellular automata, in which friction and gravity effects are taken into account in a simple way, through the parameter p. The value of the parameter p decides whether the grain will stop on the site or roll further down the slope.

The two-threshold ricepile model introduced in [6,7] is defined on a one-dimensional lattice of size L, with a wall at the zero position and an open boundary at the other end. At the open boundary, particles are free to flow out of the system.

As in the experimental setup, the system is driven by adding particles to the position setup at the closed end. Every time unit, one particle is added. Two thresholds are defined in the model: the critical threshold z_c, which is the local condition for the onset of the avalanche, and the gravity threshold z_g ($z_c < z_g$). If the local slope

$$z_i = h_i - h_{i+1} \tag{2}$$

(where h_i, $i=1,2,3,...,L$, is a height profile of pile) is less than z_c, it is too low and the friction stops the grain movement. The grain resides on the position i. If $z_i > z_g$, the local slope is too high and the grain moving downslope is not allowed to stop at the ith position. But, in the case in which $z_c < z_i < z_g$, the grain moves from i to $i+1$ with probability p. Through the parameter p, effective friction is introduced into the model. All supercritical slopes can topple, with probability p, but for local slopes, which are too big ($z_i > z_g$), the gravity becomes decisive and the site topples with probability $p=1.0$.

The dynamics of the ricepile model [6,7] is as follows.

(i) Each avalanche starts at $i=1$. If $z_1 > z_c$, the first site is activated and topples a particle to the next-nearest position (two) with the probability p. If even $z_1 > z_g$, $p=1.0$.

(ii) Every particle, sliding from the position i to $i+1$, activates three columns, namely $i-1$, i, and $i+1$. The position $i-1$ is activated because it possibly can become supercritical when removing a grain from the ith column. Columns i and $i+1$ are activated because they are destabilized by sliding or stopping particles, respectively. In the next time step, all supercritical active sites topple a particle to the $(i+1)$st position with probability p ($p=1.0$ if $z_i > z_g$).

(iii) Step two is repeated until there are no active sites in the system, which means until the avalanche is not over.

Changing slightly the local toppling rules, different versions of the ricepile model are defined [8].

(a) The number of particles toppled from the position i is constant and independent on the supercritical local slope z_i. The model is called *limited*.

(b) The number of toppled particles is a function of the supercritical local slope z_i. The model is called *unlimited*.

(c) If the particle (or more particles) topples from the site i and moves only to the next-nearest position $i+1$, the model is defined as *local*.

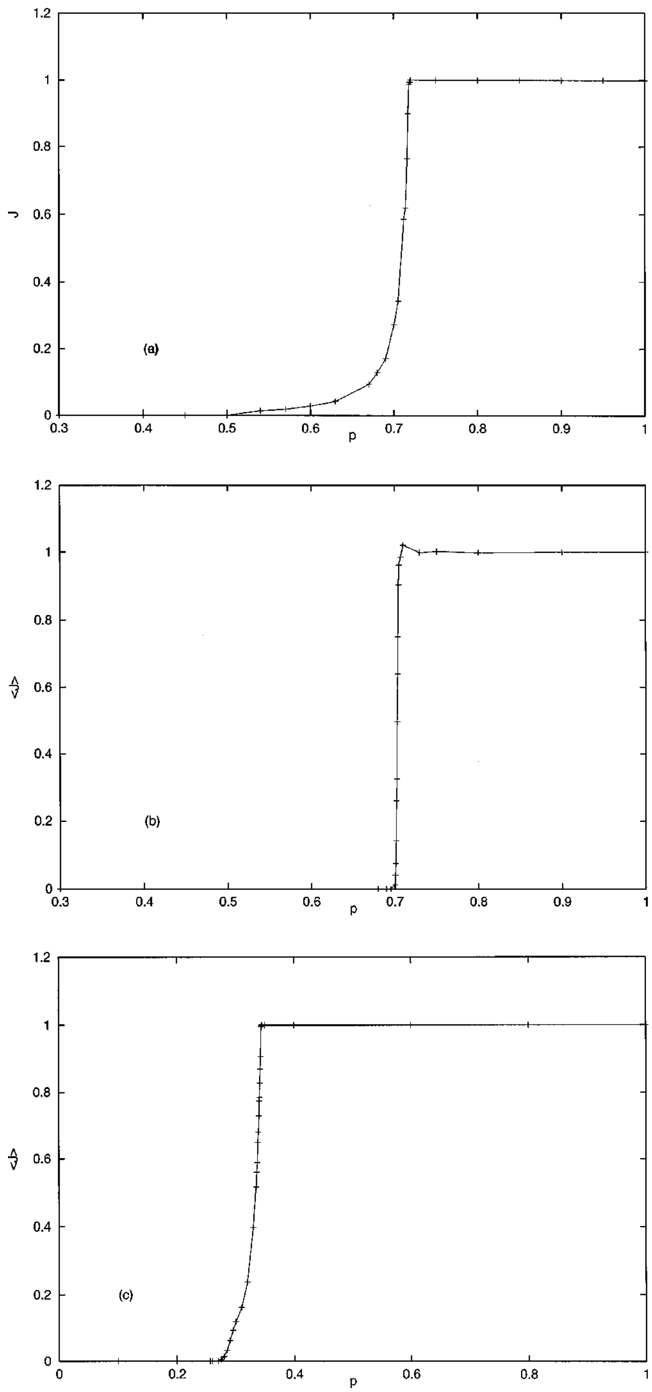

FIG. 1. Average transport ratio of particles through the system as a function of the parameter p. (a) LLIM model: Three different dynamical regimes are recognized: isolating, $p \in (0,0.538\,65)$; partially conductive, $p \in [0.538\,65,0.7185)$, and conductive, $p \in [0.7185,1.0]$. (b) LUNLIM model: Only two different dynamical regimes are recognized: isolating, $p \in (0,0.6995)$, and conductive, $p \in [0.6995,1.0]$. (c) NLIM model: Again, three different dynamical regimes are depicted: isolating, $p \in (0,0.267)$; partially conductive, $p \in [0.267,0.365)$, and totally conductive, $p \in [0.365,1.0]$.

(d) The model is called *nonlocal*, if n toppled particles moving from the ith site are added subsequently to n nearest downslope positions (one particle per site) $i+1$, $i+2$,..., $i+n$.

Thus four different ricepile models are recognized, namely the local limited model (LLIM), the local unlimited

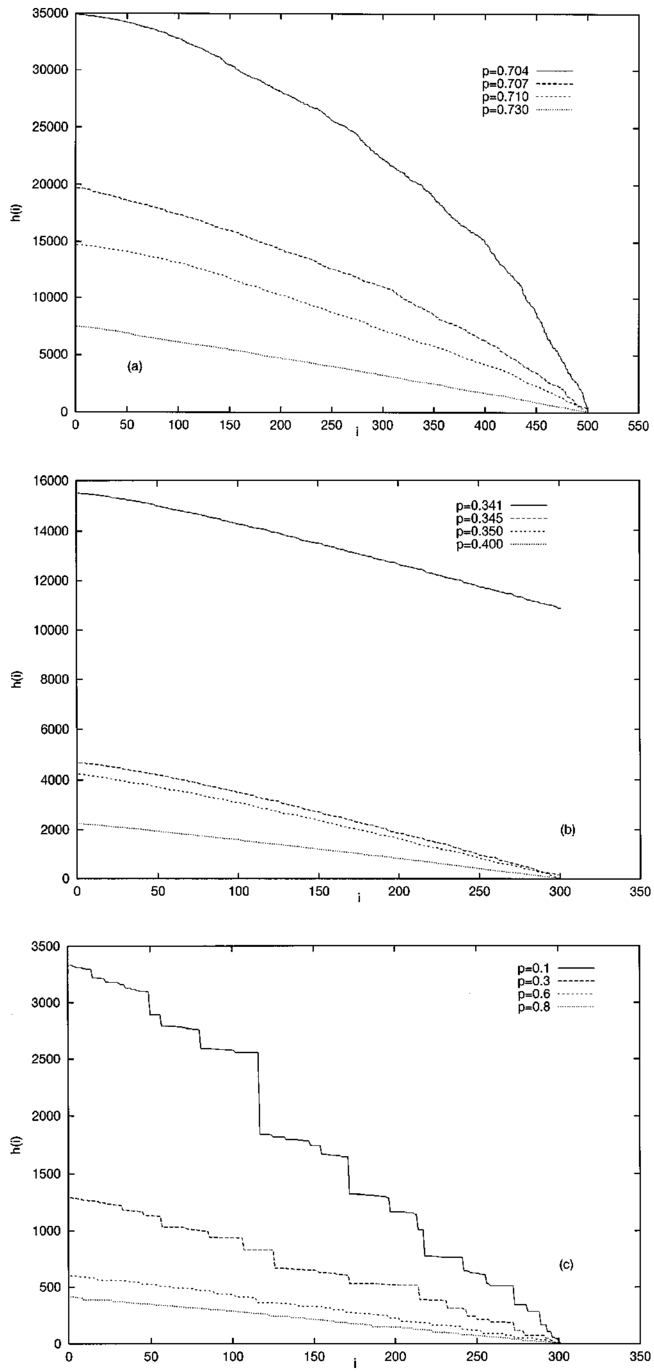

FIG. 2. Pile profiles of the LUNLIM (a), NLIM (b), and NUNLIM (c) absorbing models for different values of the probability parameter p. Notice that in the totally conductive regime, the pile profile is pinned at $i=L$ (a), (c). The pile is growing as a bulk with velocity $v(p)$ in the case of the partially conductive regime and is pinned in the totally conductive regime (b).

model (LUNLIM), the nonlocal limited model (NLIM), and the nonlocal unlimited model (NUNLIM).

Universality classes for the ricepile model were studied by Amaral and Lauritsen [8]. Their results show that local models (LLIM, LUNLIM) belong to the wide universality class called the local linear interface universality class (LLI class) [8,13]. The authors also found that the nonlocal toppling rules lead to two new universality classes, with a different set of critical exponents. But none of the universality classes is that of the real ricepile.

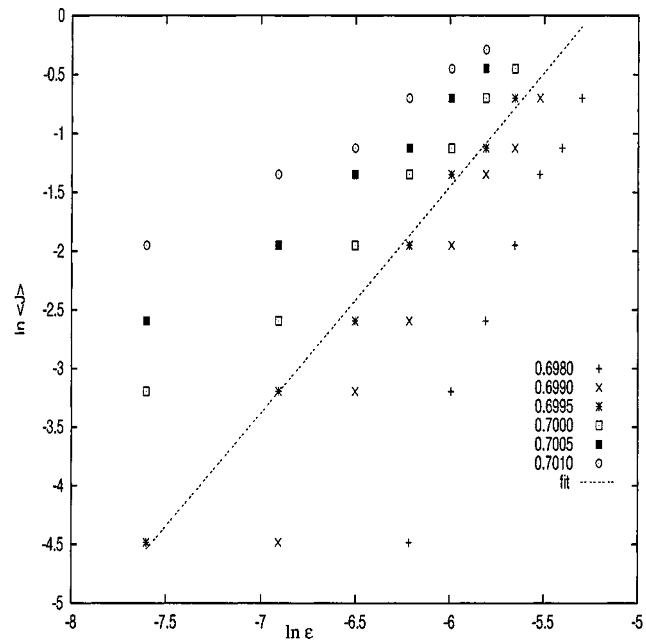

FIG. 3. ln-ln plot of the average transport as a function of the distance from the critical point p_c in the LUNLIM absorbing model, $\epsilon=(p-p_c)$. We find that the best scaling is obtained for $p_c=0.6995$.

III. ABSORBING MODEL

Our absorbing model [11] exhibits a simplified, one-threshold version of the ricepile model [6]. The gravity threshold is removed and all supercritical active sites are allowed to topple with probability $p<1$. Thus there persists a small but nonzero probability that also the extremely large local slopes are possible. Physically this seems to be quite plausible. It is not probable that in real piles of granular material there exists a strict gravity threshold. There is rather a continuous transition to the local slopes, which are already so large that, when activated, they *almost* always topple.

We investigated numerically all four versions of the absorbing model: LLIM, LUNLIM, NLIM, NUNLIM. Several quantities were measured for all of the models: (i) Material transport as a ratio of the number of outgoing to ingoing particles

$$J(p)=\frac{n_{\mathrm{out}}(p)}{n_{\mathrm{in}}(p)}, \tag{3}$$

and its dependence on the parameter p. (ii) Average material transport $\langle J(p)\rangle$ as a function of p. (iii) Avalanche size distribution for different parameter values p. Avalanche sizes are measured in terms of dissipated potential energy, in accordance with the experiment [5]. (iv) Changes in the pile profile, due to changes in the parameter p value.

If the probability parameter p changes slowly in the interval (0, 1), the model typically passes through different dynamical regimes: (i) isolating, in which all particles are absorbed in the system and none of them reaches the open boundary; (ii) partially conductive, in which the pile profile grows up as a bulk, because a certain fraction of the particles, depending on p, is absorbed in the system (*absorbing*

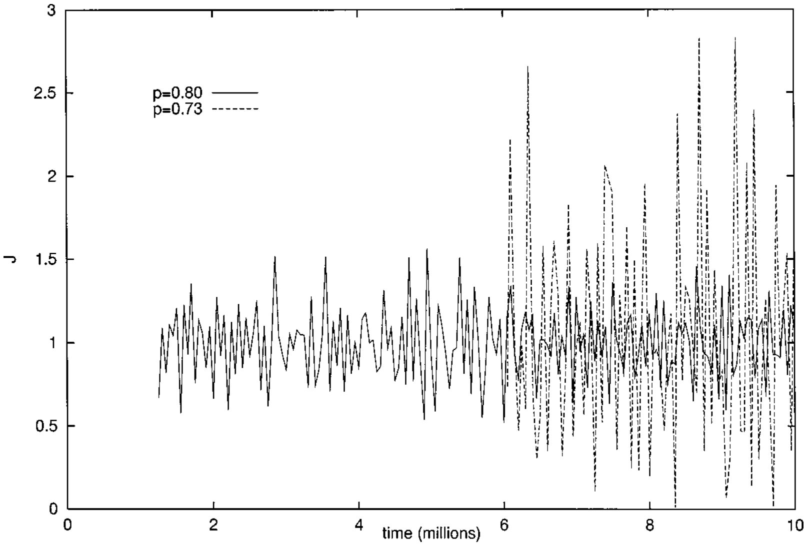

FIG. 4. Transport $J(p)$ as a function of time (in iterations) in the conductive regime of the LUNLIM model for two different values of p. For $p \geqslant p_c$, $p_c = 0.6995$, white noise is observed. For $p < p_c$, all particles are absorbed and therefore there are no fluctuations.

properties); and (iii) totally conductive, when the number of ingoing and outgoing particles is balanced.

IV. UNIVERSALITY CLASSES FOR THE ABSORBING MODEL

A. LLIM

Dynamical properties of the local limited absorbing model are described in detail in [11]. Here I only briefly list the main results.

Local limited toppling rules are defined as follows: the active supercritical site topples one particle to the next-nearest position with the probability p. Looking at the average material transport $\langle J(p) \rangle$, three dynamical regimes of the LLIM model are recognized [Fig. 1(a)].

(a) For $0 < p < p'$, $p' \approx 0.538\,65$, the system is completely isolating. The average transport $\langle J(p) \rangle$ is zero. For p close to zero, almost all ingoing particles are absorbed. The avalanches die out soon, their size being exponentially bounded.

Close to the first phase transition point p', the steepness of the pile is still high enough to say that the local slopes are almost everywhere higher than the critical threshold z_c. This is the reason that the spreading of active sites in time is practically determined by the probability p, the same way as it is in the percolation process. In the space-time coordinate system, we have therefore a picture of directed percolation with three descendants and an absorbing boundary [15]. p' is thus simply the critical percolation threshold. Close to the percolation threshold p', the average transport $J(p)$ scales with p as

$$\langle J(p) \rangle - J' \propto (p - p')^{\delta'}, \tag{4}$$

$$\delta' = 0.9 \pm 0.01,$$

where J' is the current flowing due to the finite size of the system.

(b) For the probability interval $p' < p < p_c$, $p_c \approx 0.7185$, the system is partially conductive, with constant average slope. This means that the height profile grows as a bulk with velocity $v(p)$. Fluctuations of transport $J(p)$ exhibit long-range correlations.

Above the percolation threshold p', the percolation picture breaks down. The subcritical absorbing states are randomly distributed throughout the system and the avalanche can stop anywhere. As $p \rightarrow p_c$, the long-range correlations in transport fluctuations are destroyed and the region of small local slopes spans the whole system. The pile stops to grow and at $p = p_c$ it is pinned at the position $i = L$. Critical point p_c is thus understood as the depinning transition point. Close to the depinning critical point, the average transport scales as

$$1 - \langle J(p) \rangle \propto |p - p_c|^{\delta}, \qquad \delta = 0.9 \pm 0.01. \tag{5}$$

(c) In the interval $(p_c, 1)$, the system is completely conductive. Transport fluctuations are of white noise type and the average transport $J(p) = 1.0$. In the dynamical regimes (b) and (c) the system is in the SOC state, having a power-law distribution of avalanches sizes (1) with critical exponent $\tau = 1.57 \pm 0.05$.

B. LUNLIM

Local unlimited toppling rules in the absorbing model are defined as follows: In order to get a realistic profile of the pile, each supercritical active site topples k, $k = \mathrm{int}(z_i/2.0)$, grains to the next-nearest downslope position with probability p. This way one gets a smooth profile without cavities [Fig. 2(a)].

Numerical investigations of $\langle J(p) \rangle$ reveal that only two dynamical regimes are clearly recognized [Fig. 1(b)]: the pile is either completely isolating $[\langle J(p) \rangle = 0]$ or completely conductive $[\langle J(p) \rangle = 1.0]$. A partially conductive dynamical regime is missing.

(a) For $0 < p < p_c$, $p_c \approx 0.6995$ [Fig. 1(b)], the system is completely isolating. From the definition of local toppling rules it is clear that absorbing states $(z_i < z_c)$ are easily created even for very small values of the parameter p. This means that the percolation picture in the space-time coordinate system is not correct in the case of the LUNLIM model. Near transition point p_c the average transport $J(p)$ scales with p as (Fig. 3)

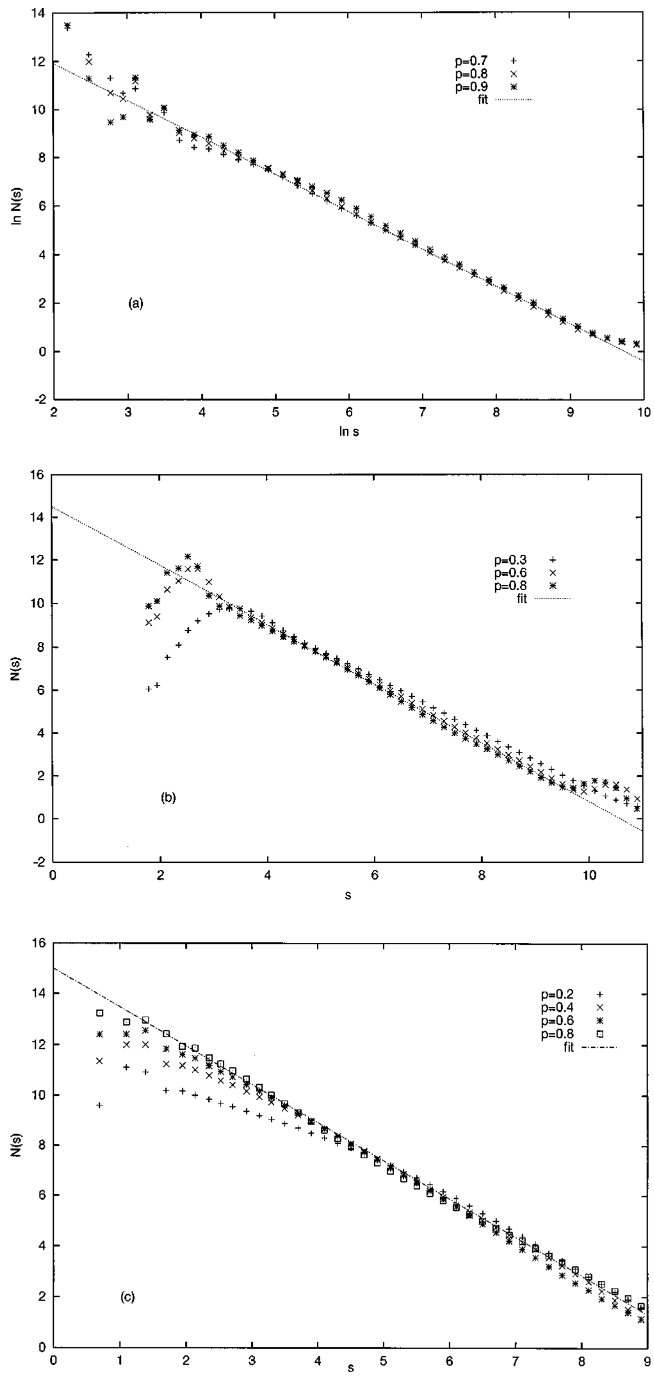

FIG. 5. ln-ln plots of the power-law parts of the avalanche size distributions (unnormalized). The critical exponent τ of the LLIM [11], LUNLIM (a), and NUNLIM (c) models is $\tau = 1.55$. The NLIM model (b) belongs to the different universality class with $\tau = 1.35$. The system size in (b) and (c) $L = 300$, and in (a) it is $L = 500$. In the NLIM model (b), the bump shift with system size has been numerically tested. With growing system size, the bump shifts to the higher values of s, indicating thus SOC.

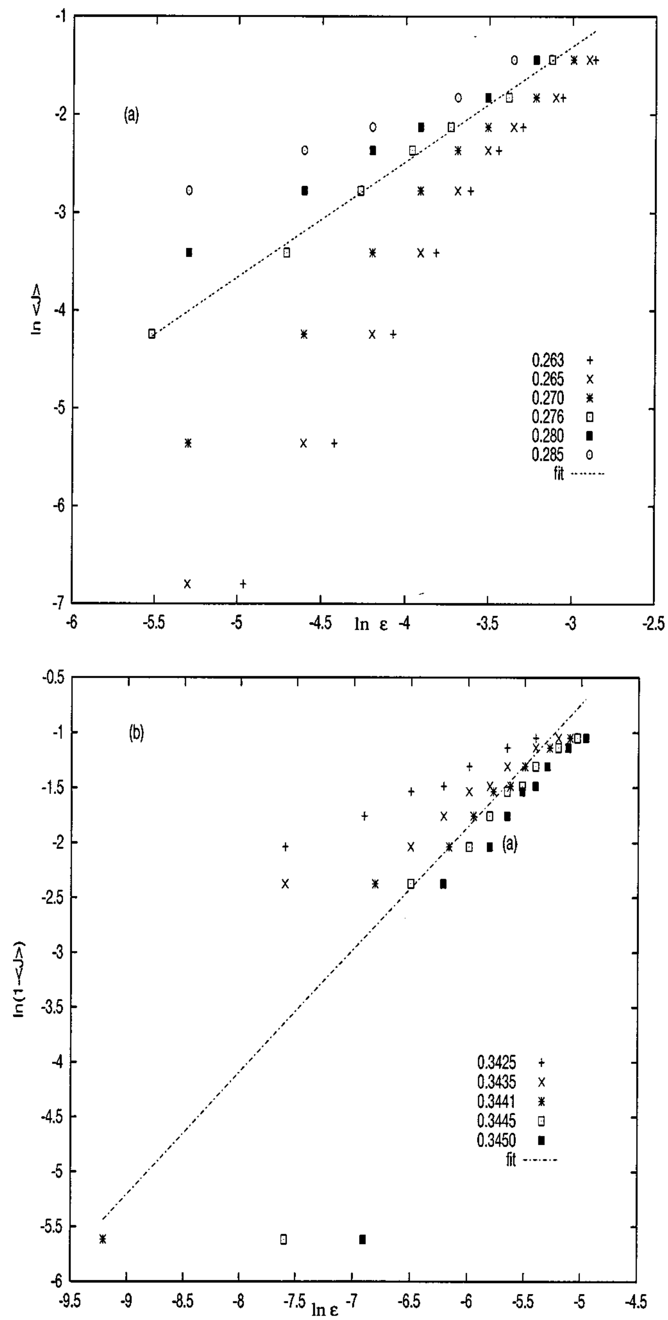

FIG. 6. (a) ln-ln plot of the average transport as a function of the distance from the critical point p' in the NLIM absorbing model, $\epsilon = p - p_c$. We find that the best scaling is obtained for $p_c = 0.276$. (b) ln-ln plot of the average transport as a function of the distance from the critical point p_c in the NLIM absorbing model, $\epsilon = p_c - p$. We find that the best scaling is obtained for $p_c = 0.3441$.

$$J(p) \propto (p - p_c)^{\delta},$$
$$\delta = 1.93 \pm 0.07. \tag{6}$$

(b) In the second dynamical regime ($p_c < p < 1.0$), the system is completely conductive, with the pile profile pinned at $i = L$ [Fig. 2(a)]. $J(p)$ as a function of time exhibits white noise features (Fig. 4). Avalanche size distribution is critical (1) with critical power-law exponent $\tau = 1.54 \pm 0.02$ [Fig. 5(a)].

C. NLIM

The nonlocal limited toppling rule means that the supercritical active site topples, with probability p, N particles to the N nearest downslope positions. The nonlocal limited toppling rules preserve three dynamical regimes, the same way

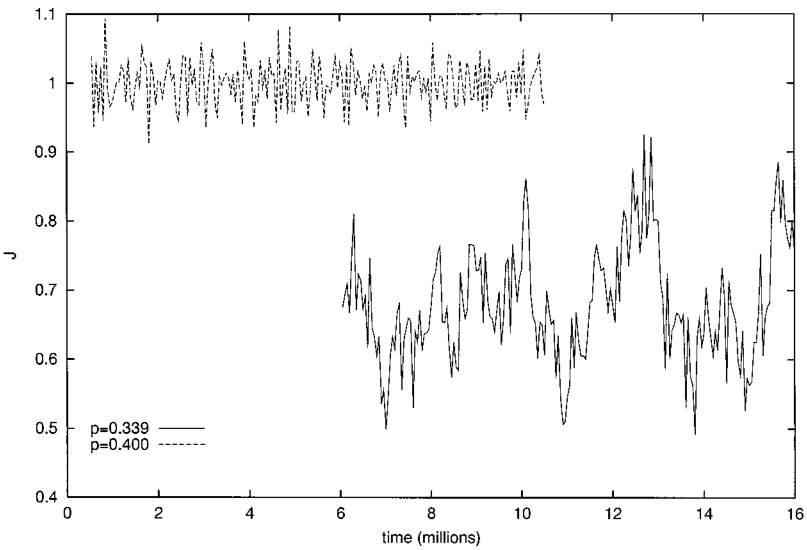

FIG. 7. Transport $J(p)$ as a function of time (in iterations) in the partially conductive regime ($p=0.339$) and totally conductive regime ($p=0.4$) of the NLIM absorbing model. For $p \geqslant p_c$, $p_c=0.365$, white noise is observed. For $p' < p < p_c$, the character of the fluctuations is different. Long-range correlations (reminiscent of Brownian motion) are observable in the time signal. For $p < p'$, all particles are absorbed and therefore there are no fluctuations.

as it is in the LLIM case. In Fig. 1(c), isolating, partially conductive, and totally conductive regimes are recognized.

(a) For $0 < p < p'$, $p' \approx 0.267$ [Fig. 1(c)], the pile is in the isolating regime. To understand the nature of the first phase transition point p', the percolation picture in the time-space coordinate system is still useful. But now the number of descendants is, in principle, greater than three. That is the reason for the fact that the percolation threshold is shifted to the lower parameter values as one can see when comparing Figs. 1(a) and 1(c), e.g., $p'_{\mathrm{NLIM}} < p'_{\mathrm{LLIM}}$. In the model studied here, the number of particles toppling from the activated supercritical site is four. Five sites are thus activated by every toppling from the position i, namely $i-1$, i, $i+1$, $i+2$, and $i+3$. There are therefore five descendant sites in the directed percolation with absorbing boundary [11]. In order to estimate p' with greater accuracy, systematic studies of the dependence of the percolation threshold on the number of descendant sites are necessary.

The average transport near the percolation threshold p' scales as [Fig. 6(a)]

$$J(p) \propto (p-p')^{\delta'} \tag{7}$$

with the critical exponent

$$\delta' = 1.18 \pm 0.04.$$

(b) In the interval $p' < p < p_c$, the pile increases with constant velocity $v(p)$, maintaining the global slope on a constant value for a constant probability parameter p. Transport $J(p)$ as a function of time shows long-range correlations, contrary to the totally conductive regime, where it has a character of white noise (Fig. 7).

(c) The depinning transition occurs at $p_c \approx 0.365$ [Fig. 6(b)]. Average transport scales with p close to the critical point as

$$1 - \langle J(p) \rangle \propto |p - p_c|^{\delta}, \qquad \delta = 1.12 \pm 0.06. \tag{8}$$

For the probability interval $p_c < p < 1$, the profile of the pile is pinned at $i = L$ [Fig. 2(b)], and the average transport $\langle J(p) \rangle = 1.0$ [Fig. 1(c)]. Figure 5(b) demonstrates the avalanche size distribution in the case of partially conductive and conductive dynamical regimes. In both cases, the dynamics of the pile is self-organized critical, which is demonstrated by the critical power-law scaling (1). The critical exponent $\tau = 1.35 \pm 0.05$.

D. NUNLIM

The nonlocal unlimited toppling rule is defined as follows: $N(z_i)$ particles are released (with probability p) from the activated supercritical position and are added to $N(z_i)$ nearest downslope positions.

The nonlocal unlimited version of the absorbing model shows completely different behavior. First, no distinct dynamical regimes are recognized. The pile is completely conductive already for p close to zero as can be seen from Fig. 8(a). The pile profile is pinned at $i = L$ [Fig. 2(c)]. Avalanche size distribution shows the critical scaling (1) with critical exponent $\tau = 1.51 \pm 0.05$ [Fig. 5(e)].

V. DISCUSSION AND CONCLUSION

The probability density function (1) scales with the system size as

$$p(s,L) = L^{-\beta} g\left(\frac{s}{L^D}\right) \tag{9}$$

with $\beta = D\tau$. For the LLIM, LUNLIM, and NUNLIM absorbing models the best data collapse has been found for $D = 2.24$, which indicates that these models belong to the same universality class, called the LLI universality class [8,13].

On the contrary, for the NLIM absorbing model the best data collapse was found for $D = 1.55$. The critical exponents τ and D are different from that of the LLI class and define a new universality class to which the NLIM version of the two-threshold ricepile model also belongs [8].

The reason for lowering the τ exponent of the NLIM model in comparison with the LLIM model is as follows: The average slope of the NLIM and the LLIM pile is similar. For example, for $p = 0.8$, the average slope of the NLIM pile

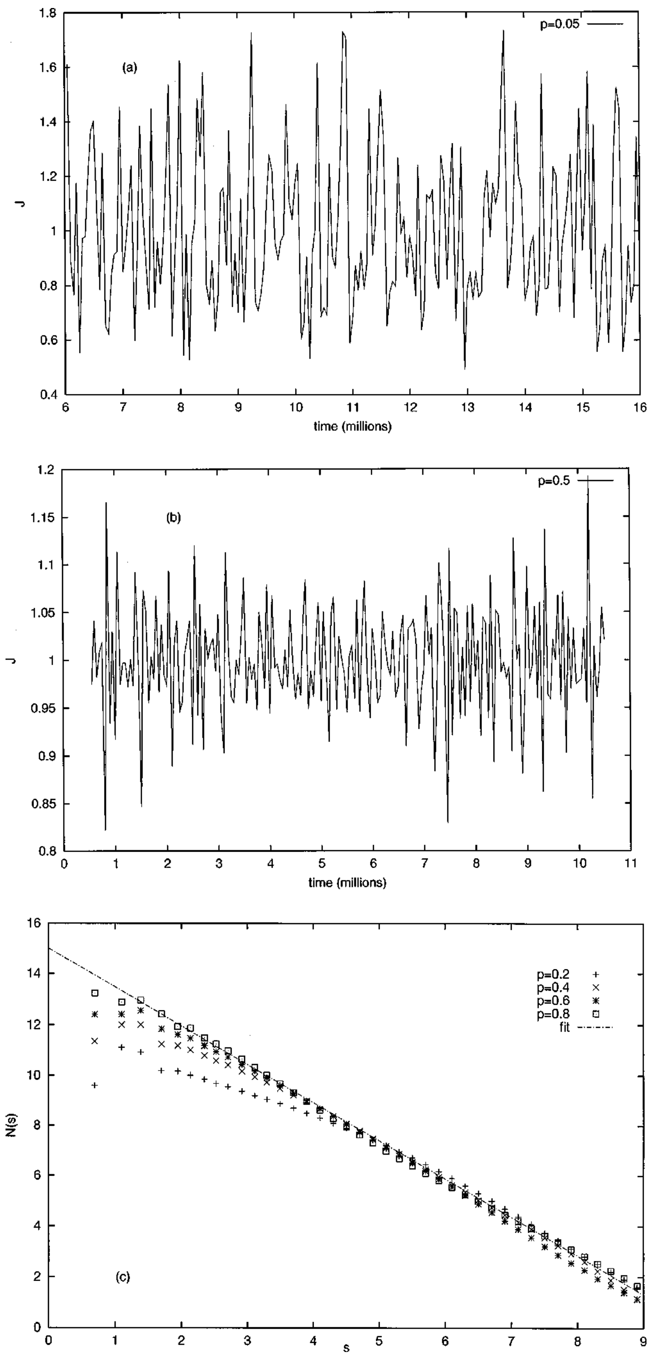

FIG. 8. Transport $J(p)$ as a function of time (in iterations) in the NUNLIM model for three different values of the parameter p. The pile is totally conductive in a wide range of the parameter p and the signal has a character of white noise, with fluctuations depending on p.

is 67.87° and for the LLIM pile it is 60.53°. Due to the nonlocal toppling rules in the NLIM model, more columns are perturbed and thus the probability of greater avalanches is enhanced. Therefore the exponent τ is lowered.

The same argument could be used in the case of the NUNLIM and the LUNLIM model. But here the situation is different. The average slope changes significantly with changes in toppling rules from local to nonlocal. For example, if $p=0.8$, the average slope of the LUNLIM model is 82.22° and that of the NUNLIM model is 53.6°. Because the number of toppled particles is proportional to the slope in the unlimited model, it seems that the relatively small average slope of the NUNLIM pile leads to relatively few particles released on average in one toppling. This fact should enhance the probability of small avalanches and the avalanche size distribution function should have a τ exponent greater than 1.55. This really happens for the NUNLIM two-threshold ricepile model [8]. In this model the probability of a big local slope decreases exponentially with z_i. But looking at [Fig. 2(c)], one can see that it is not an exception to have a big local slope in the NUNLIM absorbing pile. During a toppling event, the site with the big local slope releases a number of particles (proportional to the local slope), which disturb a lot of downslope columns. This effect increases the probability of large avalanches. It seems that in the case of the absorbing model, the two described effects balance each other and thus the exponent τ remains untouched by changed toppling rules. This is different from the NUNLIM two-threshold model [8]. Here the first effect is decisive and the model belongs to a new universality class ($\tau=1.63$).

Another question, which should be discussed, is the nonexistence of the partially conductive regime of the LUNLIM model. First, the model is local. This means that in every toppling, only three columns are activated by each toppled particle. Therefore, the probability of avalanches having a chance to reach the end of the system and thus to transport a material does not increase due to more activated sites by every toppled particle. As it has been already told, in the LLIM model there are no absorbing ($z_i<z_c$) states in the system for the isolating dynamical regime. This is not the case for the LUNLIM model. Absorbing states, therefore, exhibits another obstacle for bigger avalanches to develop and transport the particles. The existence of absorbing states, even for small parameter values, destroys the percolation picture of the spreading of active sites, and this is also the reason for the nonexisting critical percolation probability p'. Only the transition to the completely conductive dynamical regime is present.

Finally, some mention should be made of the pile slopes in all of the three dynamical regimes. In the isolating regime, the average transport $\langle J(p)\rangle=0$. All the added particles are absorbed in the system. Moreover, the avalanche sizes in this regime are exponentially bounded. This means that a majority of the particles is absorbed on the first few columns of the pile. Therefore, if the driving time t tends to infinity, the average slope of the pile grows to infinity. In consequence, this means that also the local slopes become arbitrarily large.

In the partially conductive regime, a constant amount of particles, depending on the parameter p, is absorbed in the system. As $t\rightarrow\infty$, the average slope of the pile remains constant; the pile increases as a bulk. The local slopes are finite, except for $z(L)$ [see Eq. (2)], which tends to infinity.

In the conductive regime, $\langle J(p)\rangle=1.0$. The average slope of the pile is constant depending only on the parameter p. All local slopes are finite in this regime.

In conclusion, we have studied numerically the LLIM, LUNLIM, NLIM, and the NUNLIM absorbing models. We have found that the models belong to two different universality classes, characterized by different critical exponents. Both of the universality classes are different from that of a real pile of rice. We have studied the transport properties of all of the models and found phase transitions between different dynamical regimes. We state that the dynamics of the LLIM and NLIM models is directly mapped to the dynamics of the directed percolation process at the absorbing boundary [15] for the defined interval of parameter p.

ACKNOWLEDGMENT

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